Magellan Outline

Background –

We define a Sudoku Board to be a 9x9 matrix whose entries are to be selected from the set of numbers {1,2,3,4,5,6,7,8,9}. However, these entries must be filled in based on certain constraints. No number can appear twice in the same row, column, or block, which we define as a 3x3 sub-matrix with indices {3a+1, 3a+2, 3a+3} x {3b+1, 3b+2, 3b+3} for a, b ∈ {0,1,2}. A board is matrix that follows these constraints, while a puzzle is partially filled in board. We call the entries in a puzzle givens or clues [Cooper&Kirkpatrick]. It has been proven that there are 288 equivalence classes of 4x4 Sudoku boards [Austin has this source I believe]. We also know that there are 5,472,730,538 equivalence classes of 9x9 Sudoku boards[Russell&Jarvis]. Many believe that we already know the minimal number of clues, 17, although we do not have a non-computational proof of this fact.

http://educ.jmu.edu/~taalmala/sudoku\_seriously\_galley.pdf

<http://www.afjarvis.staff.shef.ac.uk/sudoku/russell_jarvis_spec2.pdf>

Research Question –

We will be examining Sudoku boards to find each determining set. A determining set is a set of clues such that these clues provide exactly one solution to the Sudoku puzzle, or a fair solution. We will then find each critical set for each board where a critical set is defined as a minimal determining set. Given these critical sets, we will construct a histogram and examine the statistical properties of this histogram. For example, we are looking for the maximum/minimum over all the Sudoku boards’ minimum/maximum critical sets over the cardinality of the critical set. We will also take note of other distribution properties such as the mean, mode, median, range, standard deviance, and variation.

Project Goals and Objectives –

To find the answer or at least find reasonable bounds on an answer for the min, max, minimax and maxmin??

Project Impact or Significance –

Many common Sudoku puzzlers are unaware, but Sudoku has strong ties to one of modern mathematics’ major field of research, graph theory. If you consider each cell in the Sudoku matrix to be a vertex and draw an edge between vertices if they are in the same row, column, or block, then you have successfully represented a Sudoku board as a graph[need source?]. We can then imagine the Sudoku problem as a graph coloring problem. Graph coloring problems also have many valuable real-world applications, such as the famous Four Color problem or scheduling committee meetings[Sciencenews]. Surprisingly, progress made in generating theorems for these seemingly simple Sudoku puzzles may have various uses in the future as the field of graph theory continues to grow.

<https://www.sciencenews.org/article/sudoku-and-graph-theory> (valid source?)

Project Design, tasks, or methodology –

To begin to answer our question, we will first identify all the equivalence classes for 4x4 Sudoku boards. Then we will write a program (using Sage?) that loops through the set of equivalence classes and for each equivalence class we will write a clever Depth-First Search (DFS) algorithm to search through the tree of possibilities until we reach each class’ critical set. We will reduce Sudoku puzzles to a satisfiability problem to see if we have found a determining set. Given the statement in question, we will plug it into a SAT solver to determine its satisfiability. If there are zero solutions or more than one solution to the puzzle, we have gone too far. In contrast, if we have exactly one solution to the puzzle, we have found a determining set. At each point, we will save the height of the tree which represents the number of determining sets for the specific equivalence class(?).

In order to complete these steps, I need to become more familiar with graph theory, satisfiability solvers, programming in sage, etc.

Project Timeline –

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Anticipated Results/Final Products and Dissemination –

Plan to publish a paper and submit it to University of South Carolina publication, and XYZ. We also plan to present at the University of South Carolina presentation and XYZ Math Conference.

Personal Statement –

Although I am a Computer Science major, taking Discrete Mathematics with Dr. Cooper opened me up to an entirely new field of interest. I have always enjoyed my math classes, but only recently did I discover what truly comprises *mathematics*. After reading *The Man Who Loved Only Numbers: The Story of Paul Erdos*, I wanted to immerse myself in the theories and proofs of higher-order mathematics, as opposed to the computational mathematics taught throughout high school. Now, I can take a common man’s puzzle (Sudoku) and delve into the properties it has and what those properties represent in the greater scheme of mathematics.

Extra notes that I don’t want to delete

1. We find the equivalence classes (source)
2. Write a program so that we can find the critical set for each equivalence class
   1. We will do this via DFS of the tree of possibilities in addition to being clever to make things more efficient
   2. At each point we will remember the height of the tree (which represents the number of determining sets)
   3. Once we descend to the critical points we will look at the histogram
3. Then we make Sudoku puzzles a satisfiablity problem and plug into a sat solver
   1. If there are 0 solutions, we have gone too far
   2. If there is 1 solution, we have a determining set
   3. If we have more than one, we have also gone too far
4. We remember the determining sets because these are all the determining sets